STATISTICAL MODEL DEVELOPMENT OF CRATER THERMAL INERTIA FROM TES.

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Introduction: Thermal inertia of the Martian surface can be derived from brightness temperatures measured by the Mars Global Surveyor Thermal Emission Spectrometer (TES), providing information about material surface properties. We seek to understand trends in thermal inertia of impact craters by building a statistical model to include crater measurements as "explanatory variables" such as location (latitude, longitude) of the crater, crater diameter, and various ejecta morphologies. A statistical methodology is used to model and estimate the numerical values of these explanatory variables relative to thermal inertia variability.

Proper construction of a crater thermal inertia (TI) model has three major aspects: 1) Analyze the probability distribution of TI data, which is the "response variable" of the statistical model. 2) Construct a model to identify which explanatory variables relate to the variation in the TI data including a measure of crater spatial dependence. 3) Validate the ability of the model to predict crater TI values with the model's explanatory variables.

Crater Spatial Dependency: Crater location, diameter, and various ejecta morphologies are some explanatory variables of interest for modeling the behavior of crater thermal inertia as a function of the spatial distribution of the craters. Crater spatial density is constructed from a combination of the full Mars global crater spatial density distribution and a spatially random distribution for the same longitude and latitude region (0° to 360° longitude by +80° to −80° latitude, excluding the polar caps) and crater diameter distribution.

Let I_{rand} denote the spatially random crater density for the global region given above. Let I_{map} denote the actual spatial density derived from all the mapped craters of this region. The following ratio gives a probability of nonrandom crater density at any given location as

$$p(x) = \tau \frac{I_{map}(x)}{I_{map}(x) + I_{rand}(x)}, \qquad (1)$$

where $0 \leq p(x) \leq 1$ is the probability of nonrandom crater spatial density at location x, τ is a factor to scale the density ratio between 0 and 1, $I_{map}(x)$ is the mapped crater spatial density at location x, and $I_{rand}(x)$ is the spatially random density at location x. The location x is from a lattice fitted to the given Mars global region. A $p(x)$ value near 0.5 indicates spatial randomness, a probability near 0 suggests a spatially sparse region, and a probability approaching 1 indicates clustering.

TI Probability Distribution: An adequate model for the TI data depends on the probability distribution function (PDF) that best portrays the thermal inertia data. Figure 1 is a histogram of $\log_{10}$(TI) data. The histogram shows two prominent modes: one mode is $\log_{10}$ (TES) ≈ 1, and the second mode is $\log_{10}$(TES) ≈ 2.5.

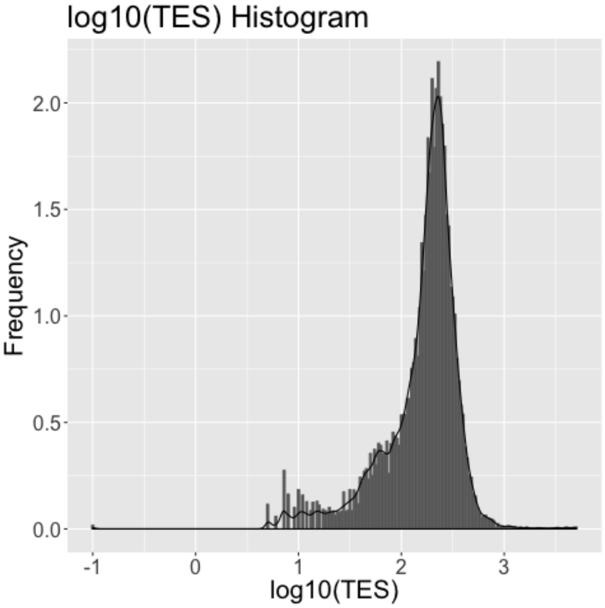

Figure 1: Histogram of $\log_{10}$(TI). There are two prominent modes.

A method to describe multi-modal PDFs is as a mixture distribution. For the bimodal TI data, let Y be the random variable $\log_{10}(TI)$ with realizations (measurements) represented as y. Then, for $f_Y(y)$ (denoting the functional representation of the mixture probability distribution) we have:

$$f_Y(y) = \phi f_{Y_1}(y_1) + (1 - \phi) f_{Y_2}(y_2), \qquad (2)$$

where the first TI component's random variable is Y_1 with measurements y_1. The second TI component's random variable is Y_2 with measurements y_2. The parameter ϕ is the fractional amount of $f_Y(y)$ due to $f_{Y_1}(y_1)$, with the second fraction due to $f_{Y_2}(y_2)$. The sum of the fractions is always 1.

Trial models of 1 to 4 components were tested and a 2-component mixture distribution gave the best model diagnostic outcomes. The first component with a Gaussian PDF and the second components with a gamma PDF produced the best TI model. Equation 3 with a Gaussian and gamma mixture has the following form:

$$\begin{aligned} f_Y(y) &= \phi f_{Y_1}(y_1|\mu,\sigma^2) + (1-\phi) f_{Y_2}(y_2|\theta,\kappa) \\ &= \phi \left[\frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(y_1-\mu)^2}{2\sigma^2} \right) \right] \\ &+ (1-\phi) \left[\frac{1}{\theta^\kappa \Gamma(\kappa)} y_2^{\kappa-1} \exp(-y_2/\theta) \right], \end{aligned} \tag{3}$$

where $-\infty < y_1 < +\infty$, $-\infty < \mu < +\infty$, $\sigma^2 > 0$, $y_2 > 0, \theta > 0$, and $\kappa > 0$. The proportion parameter ϕ is as above. As observed, $y_1 > 0$, and $\mu > 0$. The parameters μ, σ^2, θ, κ, and ϕ all are estimated using the Expected-Maximization (E-M) method [1].

t **TI Model:** The two main purposes of modeling the thermal inertia (TI) of craters are to 1) identify the significant crater morphology variables for describing TI variation, and 2) predict TI values for selected craters and their morphology variable levels. We are using TES TI data to identify trends between layered ejecta crater characteristics and thermal inertia values. The TI model represents each crater's thermal inertia by the sizes and levels of the morphology variables of these craters. Individual craters are classified by variable size and level, and an individual crater's TI behavior is modeled according to its morphological class attributes.

Equation 4 is an informal representation of the TI model:

$$\begin{aligned} \mathrm{TI} &= \mathrm{TIME{+}LAYERS{+}TERMINATE{+}EDGE} \\ &+ \mathrm{TESTYPE{+}p{+}DIAMETER{+}p} \times \mathrm{DIAMETER}. \end{aligned} \tag{4}$$

The measured response variable is the thermal inertia (TI) for each crater in the data set. The measured explanatory morphology variables specifying an individual crater's class are crater spatial density, p, and crater diameter, DIAMETER. Morphological grouping variables for each crater are LAYERS (single, double, or multiple), layer TERMINATE (pancake and rampart), and layer EDGE (broad lobe and small lobe). The type of thermal inertia measurement is TESTYPE (maximum, median, and minimum).

The size of the effect of each morphological variable is given in Table 1. The "Mixture" column is the combination of the Gaussian component and the gamma component. Each morphology variable has approximately the same effect size on TI. The "Model.TI" gives unit step changes for the categorical variables, a change of 25% in crater density, and a 10km change in crater diameter. The density by diameter interaction is a combined 25% density change and 10km diameter change. The closeness of the Mixture parameters values suggests the morphology variables have the same level of influence crater thermal inertia.

Table 1. The thermal inertia model anti-log2 of the component mixture. The parameter effect sizes (Model.TI) are in inertia units.

Variables	Mixture	Model.TI
(Intercept)	2.0659	116.3803
TIME.Night	1.0085	10.1966
LAYERS.MLE	1.0067	10.1551
LAYERS.SLE	1.0000	10.0002
TERMINATE.R	0.9961	9.9096
EDGE.SL	1.0047	10.1083
TESTYPE.Median	0.9755	9.4512
TESTYPE.Minimum	0.9152	8.2260
p	0.2396	1.7362
DIAMETER	0.9996	9.9916
I(p * DIAMETER)	0.2500	1.7785

Discussion: We constructed a statistical model to describe the crater thermal inertia behavior by classifying craters with a set of morphological variables. We used a two-component mixture probability distribution over a univariate Gaussian distribution to give crater TI values that follow the same bimodal PDF of the observed TI data. Other PDF combinations need to be examined to improve the TI mixture model outcomes.

The current model adequately reproduces TI values as expected from the TI data versus the morphological variables. The model removes much of the random variation isolating each effect's TI variability. Additional analysis areas for consideration are the randomization scheme for establishing an accurate crater spatial density variable. The choice of morphological variables, or possible non morphological variables, may need to be expanded to improve the assignment of individual craters according to the sizes and levels of these variables. Additional morphological variables should be considered. Variables constructed from combinations of the existing explanatory variables will be investigated.

While additional work is needed to refine the TI model, the statistical methodology employed in this study demonstrates that individual crater thermal inertia behavior is predictable using the crater's variables to classify thermal inertia commonalities among the individual craters, constructing that initial model.

References: [1] DeSarbo WS, Cron WL (1988). "A Maximum Likelihood Methodology for Clusterwise Linear Regression." Journal of Classification, 5, 249–282.